

# Teaching Mathematics to “Emerging Bilinguals\*” aka English Language Learners in the General Education Classroom

## What’s the difference?

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Agenda



# 5 Strategy Categories

1. Auditory/Oral
  2. Visual
  3. Instructional
  4. Scaffolded
  5. Assessment
-

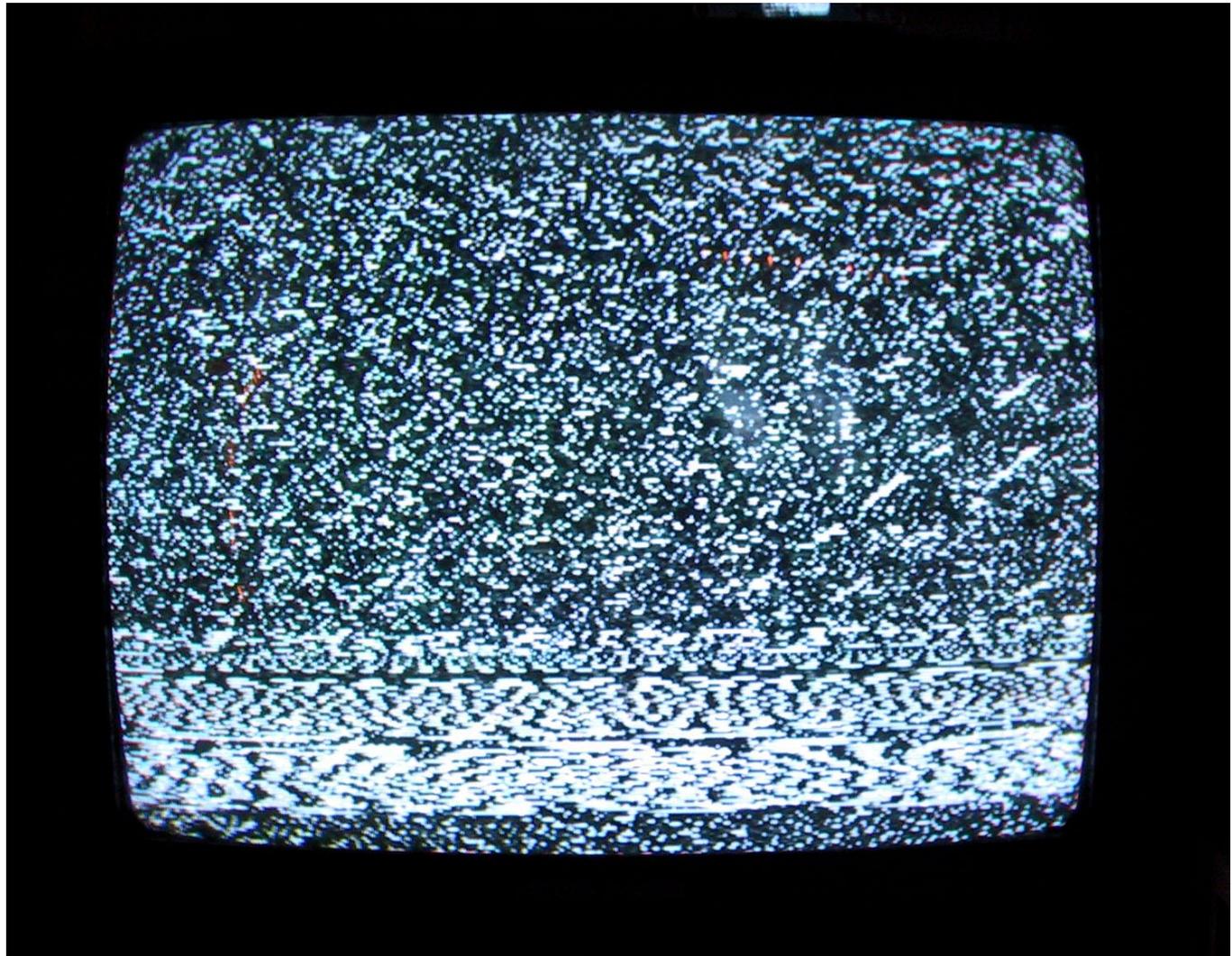
# SCENARIO

## 1

Emilio is a fourth grade student who has just moved from Mexico and is living with his grandmother. His English is broken but he can comprehend most words, especially when accompanied with gestures.

When he first arrived at school he was very excited and liked being included in the classroom atmosphere. He quickly began to withdraw and become frustrated and depressed. The teacher started to have a difficult time getting Emilio to participate in class activities.





ELLs are not necessarily  
struggling learners; they  
are learning a new  
language...

*They are becoming bilingual*



# SCENARIO

## 2

Q1. What other kinds of information should the teachers gather about Medina?

Q2. What instructional strategies might help Medina?

Q3. What might help Medina on test days?

# 5 Stages of Language Acquisition



The Short Version

# Stage 1: Pre-Production

## Minimal Comprehension with No Verbal Production

### Handout #2-I

#### Student Behavior

- Does not yet produce speech
- Listens and begins responding by using non-verbal signals
- Participates through physical actions
- Relies on picture clues for clear understanding



#### Teacher Strategies

- Surrounds students with language
- Avoids forcing students to speak prematurely
- Creates an environment that supports risk-taking
- Provides abundant opportunities for active listening using props, visuals and media



#### Teacher Prompts

- Show me ...
- Circle the ...
- Where is ...?
- Who has ...?

# Stage 2: Early Production



## Limited Comprehension with One or Two Responses

### Handout #2-1a and #2-1b

#### Student Behavior

- Responds with one or two words. →
- Repeats and recites memorable language. →
- Identifies people, places and things. →
- Understands the main idea of the message but may not understand each word. →

#### Teacher Strategies

- Uses questioning strategies eliciting one- or two-word responses.
- Uses predictable and/or patterned text.
- Asks yes/no, who, what and where questions.
- Implements shared reading with props, and builds on student prior knowledge.

#### Teacher Prompts

- Yes/no questions
- Either/or questions
- Who ...?
- What ...?
- How many ...?

# Stage 3: Speech Emergence

## Has Good Comprehension and Makes Simple Sentences (with Errors)

### Handout #2-I and #2-Ic

#### Student Behavior

- Begins speaking in phrases and short sentences. →
- Shifts the emphasis from language reception to language production. →
- Errors in grammar and syntax are common. →
- Participates more fully in discussions, including those with academic content. →

#### Teacher Strategies

- Asks questions requiring responses of phrases and short sentences.
- Models, restates, expands, and enriches student language.
- Continues to model standard language structures.
- Focuses on communication of meaningful contexts in both oral and written forms.



#### Teacher Prompts

- Why ...?
- How ...?
- Explain ...
- Questions requiring phrase or short-sentence answers

# Stage 4: Intermediate Language Proficiency

Good to Excellent Comprehension with Grammatical Errors

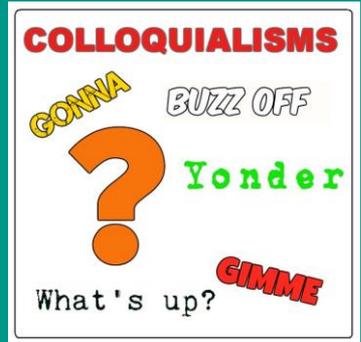
Handout #2-I and #2-Id

## Student Behavior

- Responds with a flow of related phrases and sentences. →
- Uses more extensive vocabulary. →
- Explores concepts in subject matter more extensively. →

## Teacher Strategies

- Explicitly teaches more complex grammatical forms.
- Introduces colloquialisms and idiomatic expressions.
- Guides students' use of reference materials for research purposes.



## Teacher Prompts

- What would happen if ...?
- Why do you think ...?
- Questions requiring more than a sentence response

# Stage 5: Advanced Language Proficiency

## Excellent Comprehension with Complex Speech Patterns, Few Grammatical Errors

### Handout #2-I and #2-Ie

#### Student Behavior

- Near native speech fluency — uses grammar and fluency comparable to that of same-age native speakers
- Expanded vocabulary to succeed academically
- Very good comprehension of information



#### Teacher Strategies

- Allows students to lead group discussions
- Encourages independent use of reference materials and technology
- Provides explicit grammar instruction
- Provides opportunities for student-generated presentations
- Provides a variety of realistic writing opportunities in a variety of genre



#### Teacher Prompts

- Decide if ...
- Retell ...

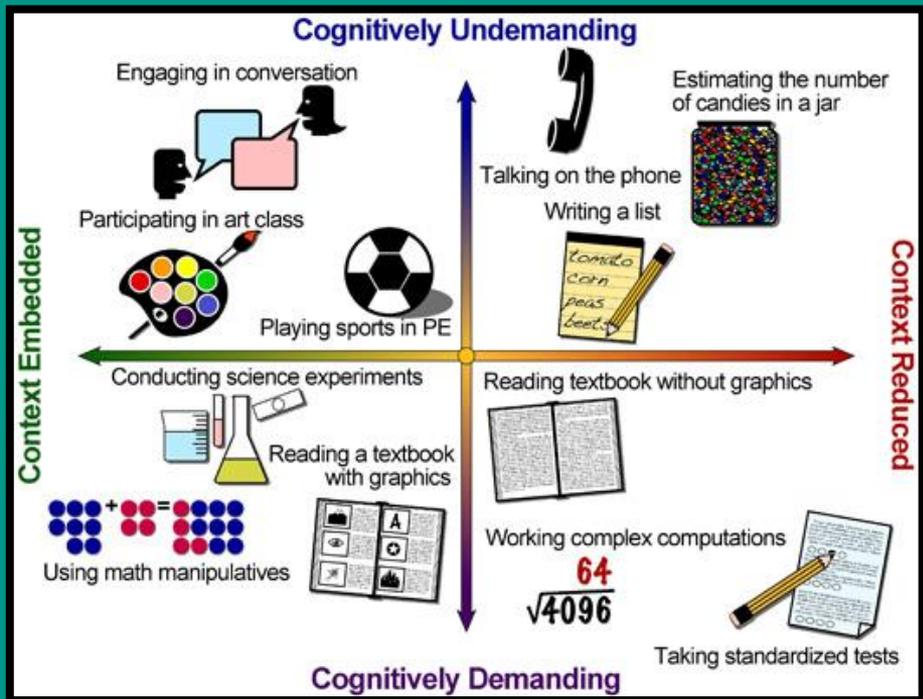
# CULTURE SHOCK.

## Stages of culture shock

The five stages of culture shock are:

1. **The Honeymoon Stage** - You are very positive, curious, and anticipate new exciting experiences. You even idealize the host culture.
2. **Irritability and Hostility** - You start to feel that what is different is actually inferior. The host culture is confusing or the systems are frustrating. It's a small step from saying that they do things in a different way to saying that they do things in a stupid way. You may blame your frustrations on the new culture (and its shortcomings) rather than on the adaptation process.
3. **Gradual Adjustment** - You feel more relaxed and develop a more balanced, objective view of your experience.
4. **Adaptation of Biculturalism** - You feel a new sense of belonging and sensitivity to the host culture.
5. **Re-entry Shock** - You go home and it isn't what you expected it to be.

# CUMMINS' QUADRANTS



**BICS are cognitively undemanding activities**

		A	C
Context Embedded		Following directions Face-to-face conversation Buying lunch at school Music, Art, PE classes	Note on the refrigerator Written directions (no visuals) Telephone conversation Oral presentation
		Demonstrations/Experiments Audio-visual assisted lessons Basic math computations Projects and activities Making models/charts/graphs	Standardized Tests Reading/Writing in content areas Math concepts and applications Lecture with few illustrations Textbooks
		B	D

**... and CALP are cognitively demanding**

**BICS are cognitively undemanding activities**

	A	C	
	Following directions Face-to-face conversation Buying lunch at school Music, Art, PE classes	Note on the refrigerator Written directions (no visuals) Telephone conversation Oral presentation	
Context Embedded			Context Reduced
	B	D	
	Demonstrations/Experiments Audio-visual assisted lessons Basic math computations Projects and activities Making models/charts/graphs	Standardized Tests Reading/Writing in content areas Math concepts and applications Lecture with few illustrations Textbooks	

**... and CALP are cognitively demanding**

Table 1. Examples of Mathematical Activities in Cummins' Quadrant Model

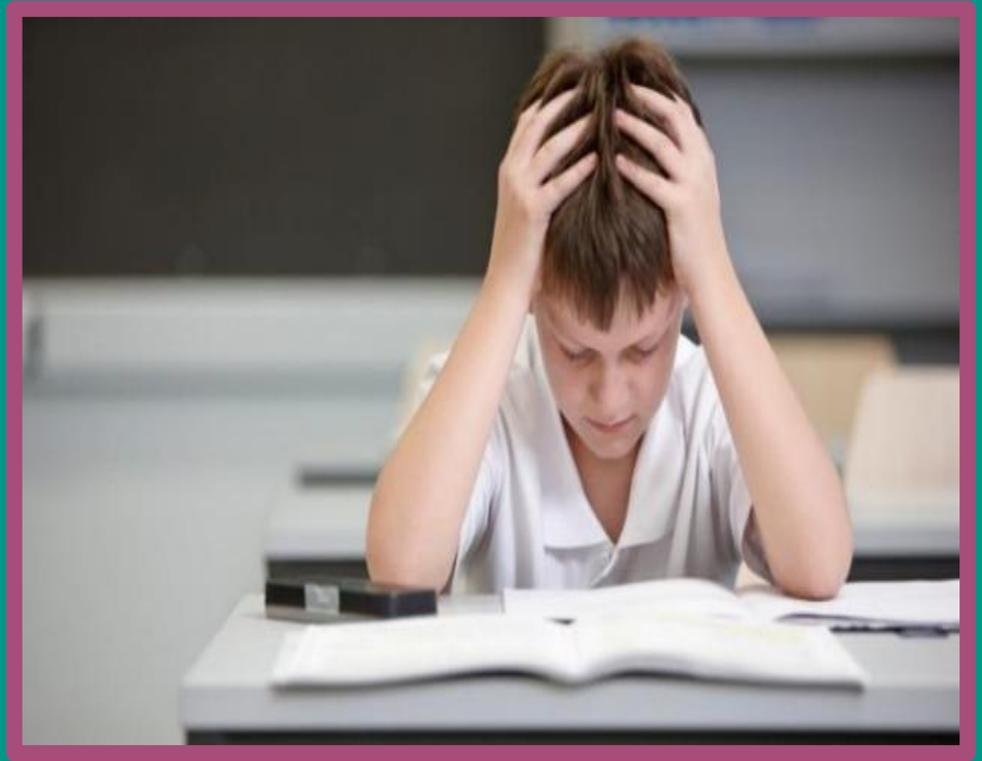
Quadrant	Example
A	Introduce geometrical shapes with real objects
B	Solve linear equations with one variable using weighing experiments Design a house with various ratios and proportions
C	Practice simple operations, Memorize times tables
D	Lecture about the properties of isosceles triangles, Algebraic proof

# Return to the Scenario 1

Emilio is a fourth grade student who has just moved from Mexico and is living with his grandmother. His English is broken but he can comprehend most words, especially when accompanied with gestures [REDACTED]. When he first arrived at school he was very excited [REDACTED] and liked being included in the classroom atmosphere. He quickly began to withdraw and become frustrated and depressed [REDACTED]

[REDACTED] The teacher started to have a difficult time getting Emilio to participate in class activities.

# The Problem with WORD PROBLEMS for ELLs



# Imagine this...



Ratio analysis

Fixed interest securities

Bulk

Facultative reinsurance

Deferred annuity

Broker-deal

Convertible

## Language of Mathematics

<u>Addition Key Words</u>	<u>Subtraction Key Words</u>	<u>Multiplication Key Words</u>	<u>Division Key Words</u>	<u>Equals Key Words</u>
sum	less (than)	multiplied	divided (evenly)	is/are
plus	minus	times	quotient	the same as
add	decrease or decreased by	total	split	equals
and	more than	of	each	equal to
total (of)	difference	per	cut	result is
increased by *	reduce	as much	equal pieces	
more (than)	lost	twice	average	
raise	left	by	every	
combined	remain (der) (ing)	area	out of	
in all	fell	volume	ratio	
altogether	dropped	product	shared	
additional	change	apiece	how many times	
together	nearer	doubled	shared equally	
both	farther	tripled	per	
added to	how much more		part	
	how much less			
	diminished			
	grow down			
	increase *			
	subtract from			

# *Ellipsis*

## *Example: 1*

*All numbers greater than six... is short for*

*All numbers (**that are**) greater than six*

## *Example: 2*

*John earns twice as much money as*

*Robert is short for*

*John earns twice as much money as Robert (**earns**)*

# *Pronouns*

*Juan has 16 action figures. He gave 9 of them away. How many action figures does he have now?*

## ORIGINAL PROBLEM:

If Peter can type one page of his homework report on his computer in 17 minutes, how much time will it take home to type three pages?

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If Peter can type one page of his homework report on his computer in 17 minutes, how much time will it take him to type three pages?

## REWRITTEN PROBLEM:

How much time will it take Peter to type three pages? He can type one page in 17 minutes.

# Put the Question first!

## Quantitative Relationships:

Hardly, higher, last, least, less, longer, many, most, next, order, rarely, scarcely, and younger

## Logical Relationships:

Alike, almost, always, because, different from, exactly, if, never, not quite, opposite, probably, same, since, unless, and whether

Relationship  
PROBLEM  
Examples:

Billy is *as old as* Jane

Jane is 6 years *younger than* Robert

All numbers *greater/less* than x

By *what percent* is 12 increased to make 18?

*Divided by* versus *Divided into*

*Divided by*(2 into 4=.5) *Divided into*(2 goes into 4 how many times=2)

Bart earns five times *as much as* Homer

When 8 is *added* (passive voice) to x

## PROBLEM:

Apple Valley School is having a big fair. The school custodian wants to buy enough sod for the field where the fair will take place. The field is 100 feet long and 75 feet wide. How many square feet is the field that he will need to cover?

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With the Question first:

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HOW MANY SQUARE FEET OF **SOD** IS NEEDED? The field is 100 ft long and 75 feet wide.

Or what is the area of the field if is 100 feet long and 75 feet wide?

## PROBLEM:

Apple Valley School is having a big fair. The school custodian wants to buy enough sod for the field where the fair will take place. The field is 100 feet long and 75 feet wide. How many square feet is the field that he will need to cover?

## Questions to consider

- Is there extra information that is not needed to solve the problem?
  - If so, then cross it out.
- Clarify vocabulary:
  - Draw pictures of the vocabulary words

# A Successful Strategy to approach Word Problems

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STUDENTS ARE TO:

1. READ THE PROBLEM TWICE
2. TRANSLATE THE PROBLEM INTO SPANISH or the student's language if possible.
3. INFER MEANING (relationship words for example)
4. USE SYMBOLS TO HELP THEM UNDERSTAND THE MATH
5. IGNORE IRRELEVANT WORDS

# Academic Language of Mathematics

Justify, evaluate, compare, contrast, summarize, interpret, state in your own words, work backwards, state a conclusion, list, explain, paraphrase, share with a partner, define, etc



## Pre-K to Grade 2-All Students should...

Understand situations that entail **multiplication** and **division**, such as **equal groupings** of objects and sharing equally.

## Grades 6-8, All Students should...

Compare and order **fractions**, **decimals**, and **percents** efficiently and find their approximate locations on the **number line**

Be Cognizant of the language in your objective(s). What needs to be explained, what needs to be simplified, what other words can I use when teaching to this objective?

## Grades 9-12-All Students should...

- Use **number theory** arguments to justify relationships involving **whole numbers**.

Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.

# 5 PRINCIPLES

## for creating effective second language acquisition-rich learning environments:

**Principle 1:** Give ELLs many opportunities to read, to write, to listen to, and to discuss Oral and Written English and Mathematics texts expressed in a variety of ways (Academic literacy)

**Principle 2:** Draw attention to patterns of English and Mathematics language structure

**Principle 3:** Give ELLs classroom time to use their English productively while learning mathematics

**Principle 4:** Give ELLs opportunities to Notice their errors and to correct their English while learning mathematics

**Principle 5:** Construct activities that maximize opportunities for ELLs to interact with others in English

# AUDITORY/ORAL

"Another perspective  
to consider is..."

"My position on  
that issue is..."

"Do you have evidence  
to support your  
position?"

# QUESTIONING



## Question Matrix

	<i>Event</i>	<i>Situation</i>	<i>Choice</i>	<i>Person</i>	<i>Reason</i>	<i>Means</i>
	I. Knowledge			II. Comprehension Application		
<b>Present</b>	1. What Is?	2. Where/ When Is?	3. Which Is?	4. Who Is?	5. Why Is?	6. How Is?
<b>Past</b>	7. What Did?	8. Where/ When Did?	9. Which Did?	10. Who Did?	11. Why Did?	12. How Did?
<b>Possibility</b>	13. What Can?	14. Where/ When Can?	15. Which Can?	16. Who Can?	17. Why Can?	18. How Can?
<b>Probability</b>	19. What Would?	20. Where/ When Would?	21. Which Would?	22. Who Would?	23. Why Would?	24. How Would?
<b>Predictability</b>	25. What Will?	26. Where/ When Will?	27. Which Will?	28. Who Will?	29. Why Will?	30. How Will?
<b>Imagination</b>	31. What Might?	32. Where/ When Might?	33. Which Might?	34. Who Might?	35. Why Might?	36. How Might?
	III. Synthesis & Analysis			IV. Synthesis & Analysis		

From Dr. Chuck Wiederhold's book *Cooperative Learning & Higher Level Thinking*, available from Kagan Cooperative Learning 27134 Espada, Suite 303, San Juan Capistrano, CA - 1-800-933-2667.

Arth, M. A. Teacher questioning in the elementary mathematics classroom: Engaging students in an on-task dialogue. University of Wyoming. Retrieved from Dissertation

# CHORAL READING In Math?

Choral reading gives ELLs the opportunity to **try out language**. It also aids the students in **improving** their sight vocabulary and **develop** oral language skills. ELLs are more willing to participate because they aren't in the spotlight.



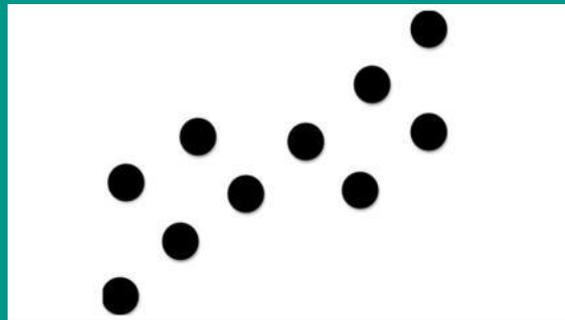
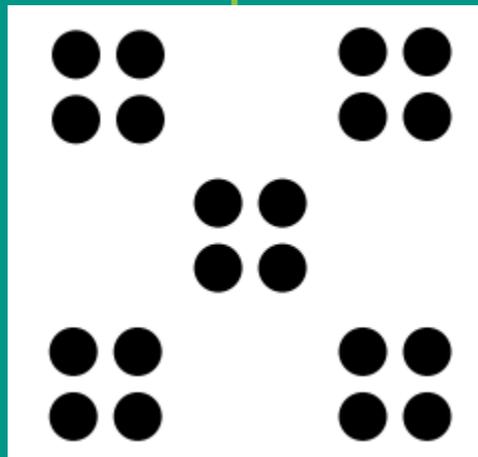
An obtuse angle is an angle that is between 90 and 180 degrees.



# Math Talks

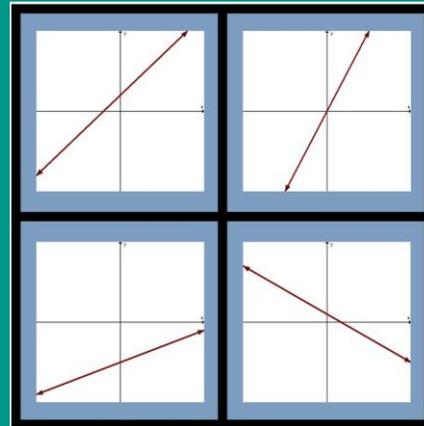
When we don't give students the time to process, negotiate for meaning, and collaborate, we have to wonder... are we supporting them as learners?

# Dot Talks

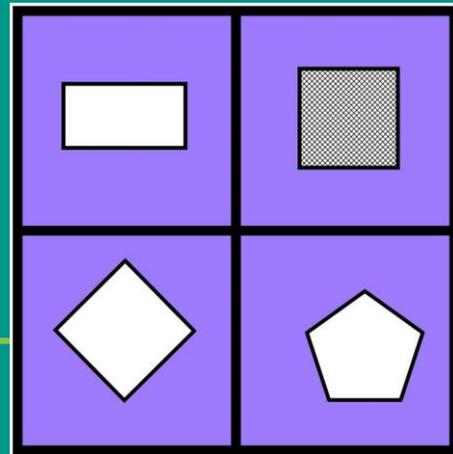


# Which One Doesn't Belong?

$(2x + 1)(x + 5)$	$(x^2 + 1)(x + 5)$
$(t + 1)(t + 5)$	$x(x+5)$

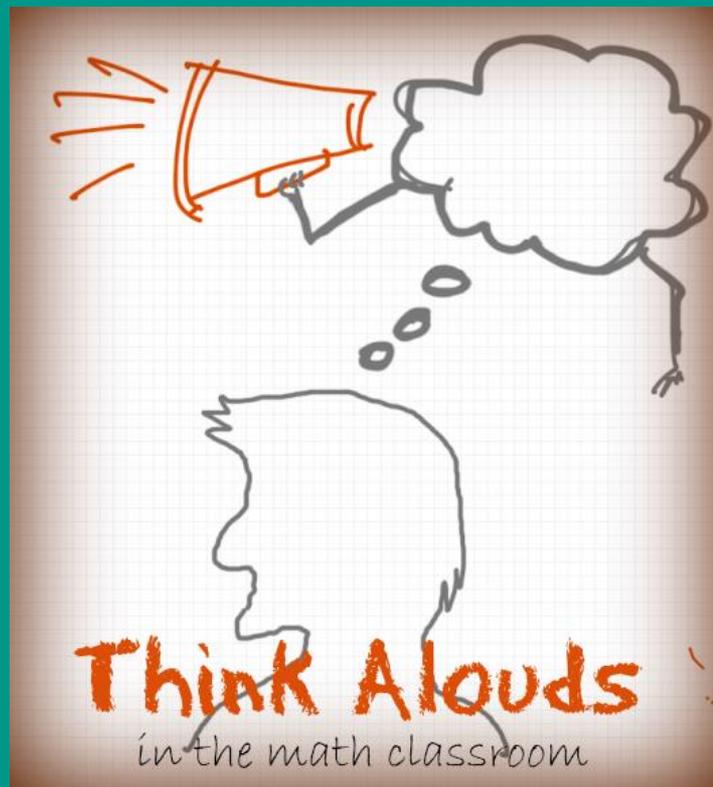


$\frac{1}{2}$	$\frac{5}{3}$
$\frac{2}{10}$	$\frac{2}{5}$

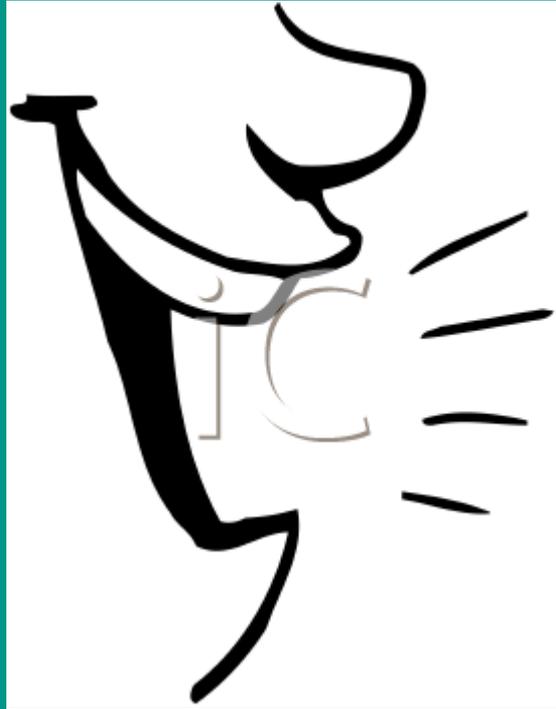


# THINK-ALOUDS

This strategy helps students see the strategies and the language that the teacher applies to solve a problem. But more importantly, Think-Alouds allow the teacher to hear how the students approach solving a problem.



# DISCOURSE IN THE CLASSROOM!



Eyes on Math

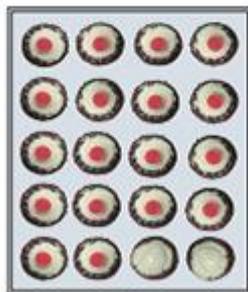
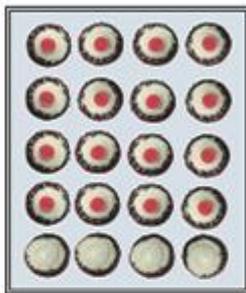
3 Act Tasks

sample

SPLAT!

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What does each picture show  
about addition?



ADDITION TO DESCRIBE PART-PART-WHOLE SITUATIONS • Grades K–2 • CCSS 1.OA

From *Eyes on Math: A Visual Approach to Teaching Math Concepts* by Marian Small. Illustrations by Amy Lin.  
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# IRE AND IRF

One of the most important features of all classroom discourse is that it **follows** a fairly typical and predictable structure, comprising three parts:

1. a teacher Initiation,
2. a student Response, and,
3. a teacher Feedback,

commonly known as IRF, or IRE: Initiation, Response, Feedback/ Evaluation.



# VISUAL STRATEGIES



# VISUALS

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This strategy helps students see the strategies and the language that the teacher applies to solve a problem. Visuals are powerful tools for ELLs in all content areas.

## Sentence Frames

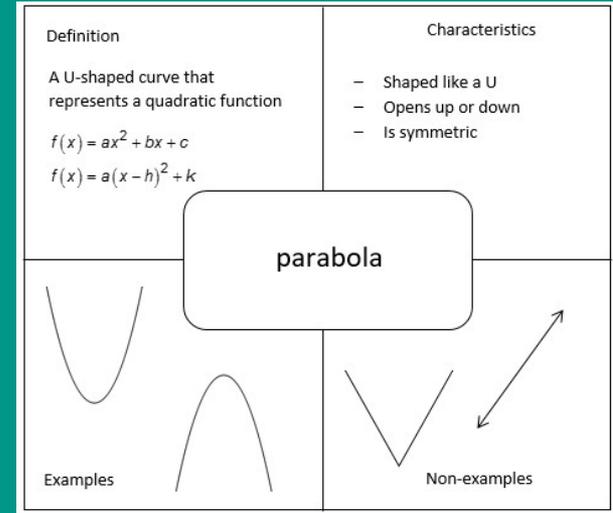
Math Modeling

Graphic Organizers

Videos

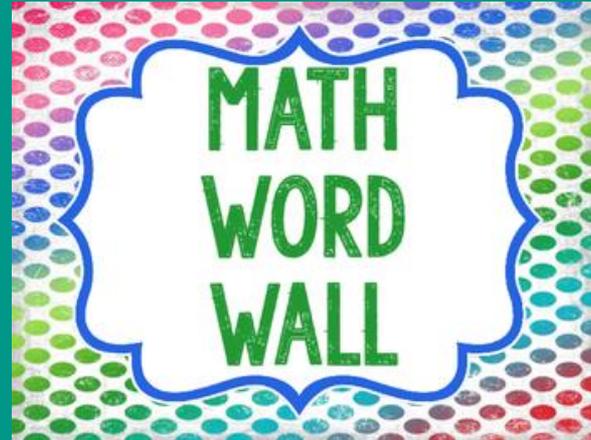
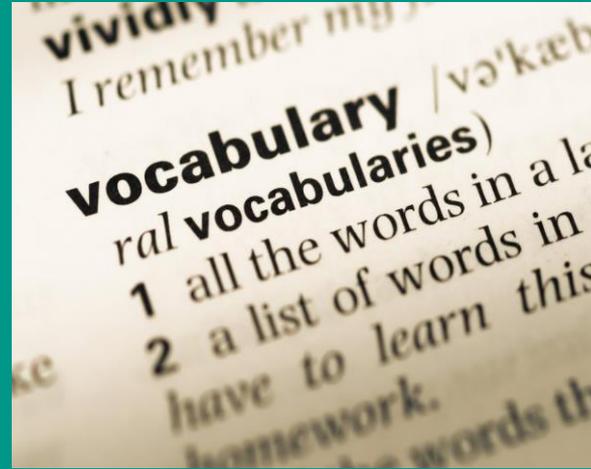
# Graphic Organizers

Graphic organizers are a great tool to use when teaching English language learners (ELLs). Visual illustrations allow ELLs to better understand the material while learning important vocabulary.





# Interactive Dictionary



- Teach Vocabulary explicitly and reinforce it on an ongoing basis.
- Post mathematical symbols with word definitions and examples to clarify meaning.

# INSTRUCTIONAL STRATEGIES



# OVERVIEW



1. Simplify Language (no slang, speak slower, don't ramble)
2. Break assignments into segments
3. Check for comprehension frequently
4. Model expectations/finished product
5. Allow some use of native language (as a tool for accessing English)
6. Extended time for classwork
7. Modified grading (language vs. content)
8. Use Multiple modes of presenting information (visual, written, oral)
9. Align with/refer to learning objectives
10. Maintain consistent routines

- Ask students to restate other students' comments, ask a question, or add their own ideas;
- When showing the steps of problem solving, explain your thought process (**think aloud**) by using vocabulary that English learners will understand;
- Provide **sentence prompts** for ELLs to use as they learn new academic language in mathematics;
- Help students through the use manipulatives, interactive games, and technology as thinking tools;
- Encourage ELLs to use graphic organizers, diagrams, gestures, and sketches to aid in thinking and communicating with others and with you;
- Correct errors and give positive feedback using prompts, gestures, encouragement, and praise.
- Provide handouts that help students structure, organize, and guide their work such as **skeleton notes**, summaries of past concepts, and outlines of future topics to be covered



A growing body of research in general supports the fact that when schools view linguistic and cultural diversity as a resource rather than a deficiency and **hold high expectations for Latino/a students**, they more times than not experience consistent academic growth in achievement



THANK YOU!

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